Test 3 Review Sheet Answers

Integer Addition and Subtraction (Chip Model)

1. We put 5 negafish and 2 posifish in the tank. 2 pairs of negafish and posifish obliterate each other, leaving 3 negafish.
2. We put 2 negafish and 4 more negafish in the tank. We now have 6 negafish in the tank.
3. We put 3 posifish and 1 negafish in the tank. 1 pair of posifish and negafish obliterate each other, leaving 2 posifish.
4. We put 4 posifish and 4 negafish in the tank. All 4 pairs of posifish and negafish obliterate each other, leaving nothing in the tank.
5. We put 2 posifish in the tank and need to remove 6 posifish. So, we create 4 pairs of posifish and negafish, giving us 6 posifish and 4 negafish. We then remove the 6 posifish, leaving 4 negafish.
6. We put 3 negafish in the tank and need to remove 5 negafish. So, we create 2 pairs of posifish and negafish, giving us 2 posifish and 5 negafish. We then remove the 5 negafish, leaving 2 posifish.
7. We put 2 negafish in the tank and need to remove 2 posifish. So, we create 2 pairs of posifish and negafish, giving us 2 posifish and 4 negafish. We then remove the 2 posifish, leaving 4 negafish.
8. We put 3 posifish in the tank and need to remove 1 negafish. So, we create 1 pair of posifish and negafish, giving us 4 posifish and 1 negafish. We then remove the 1 negafish, leaving 4 posifish.
9. We put nothing in the tank and need to remove 5 negafish. So, we create 5 pairs of posifish and negafish, giving us 5 posifish and 5 negafish. We then remove the 5 negafish, leaving 5 posifish.

Integer Addition and Subtraction (Number Line Model)

1. We stand on $^{-}5$, face right, then move forward 2, leaving us at $^{-}3$.
2. We stand on $^{-}2$, face right, then move backwards 4, leaving us at $^{-}6$.
3. We stand on 3, face right, then move backwards 1, leaving us at 2.
4. We stand on 4, race right, then move backwards 4, leaving us at 0.
5. We stand on 2, face left, then move forward 6, leaving us at $^{-}4$.
6. We stand on $^{-}3$, face left, then move backwards 5, leaving us at 2.
7. We stand on $^{-}2$, face left, then move forward 2, leaving us at $^{-}4$.
8. We stand on 3, face left, then move backwards 1, leaving us at 4.
9. We stand on 0, face left, then move backwards 5, leaving us at 5.

Integer Multiplication (Chip Model)

1. We put 3 groups of 2 posifish in the tank, giving us 6 posifish total.
2. We put 3 groups of 2 posifish in the tank. Then, since 3 is negative, we flip everything in the tank, giving us 3 groups of 2 negafish in the tank. We have 6 negafish total.
3. We put 3 groups of 2 negafish in the tank, giving us 6 negafish total.
4. We put 3 groups of 2 negafish in the tank. Then, since 3 is negative, we flip everything in the tank, giving us 3 groups of 2 posifish in the tank. We have 6 posifish total.
5. We make no groups, leaving us with nothing in the tank.
6. We make 4 groups of nothing, leaving us with nothing in the tank.

Integer Multiplication (Number Line Model)

1. We start at 0, face right, and move forward 3 times with steps of length 2, leaving us at 6.
2. We start at 0, face left, and move forward 3 times with steps of length 2, leaving us at $^{-}6$.
3. We start at 0, face right, and move backwards 3 times with steps of length 2, leaving us at $^{-}6$.
4. We start at 0, face left, and move backwards 3 times with steps of length 2, leaving us at 6.

5. We start at 0, face right (or left), and move backwards no times with steps of length 1. We stay at 0.
6. We start at 0, face right and move forwards (or backwards) 4 times with steps of length 0. We stay at 0.

Building and Simplifying Fractions

1. $\frac{3}{12} = \frac{1}{4}, \frac{1}{4} = \frac{2}{8} = \frac{4}{16}$
2. $\frac{26}{52} = \frac{1}{2}, \frac{1}{2} = \frac{2}{4} = \frac{3}{6}$
3. $\frac{42}{60} = \frac{7}{10}, \frac{7}{10} = \frac{14}{20} = \frac{21}{30}$
4. $\frac{252}{78} = \frac{42}{13}, \frac{42}{13} = \frac{84}{26} = \frac{126}{39}$
5. $\frac{616}{96} = \frac{77}{12}, \frac{77}{12} = \frac{154}{24} = \frac{231}{36}$
6. $\frac{2662}{165} = \frac{242}{15}, \frac{242}{15} = \frac{484}{30} = \frac{726}{45}$

Comparing Fractions

1. Make a common denominator of 15: $\frac{5}{15} < \frac{6}{15}$, so $\frac{1}{3} < \frac{2}{5}$.
2. Simplify to $\frac{4}{5}$ ___ $\frac{8}{11}$. Not equal, so try cross multiplication: $4 \cdot 11 = 44 > 5 \cdot 8 = 40$, so $\frac{16}{20} > \frac{8}{11}$.
3. Simplify to $\frac{2}{15}$ ___ $\frac{2}{15}$. These are equal, so $\frac{4}{30} = \frac{18}{135}$.
4. Make a common denominator of 70: $\frac{36}{70} > \frac{35}{70}$, so $\frac{18}{35} > \frac{1}{2}$.
5. Cross Multiply: $7 \cdot 23 = 161 > 11 \cdot 12 = 132$, so $\frac{7}{11} > \frac{12}{23}$.
6. Cross Multiply: $47 \cdot 23 = 1081 > 999 = 9 \cdot 111$, so $\frac{47}{111} > \frac{9}{23}$.

Proofs 1

1. The following steps can be performed forwards or backwards:
$$\frac{a}{b} = \frac{c}{d}, \quad \frac{a \cdot d}{b \cdot d} = \frac{c \cdot b}{d \cdot b}, \quad \frac{ab}{bd} = \frac{bc}{bd}, \quad ad = bc. \ \blacksquare$$
2. The following steps can be performed forwards or backwards:
$$\frac{a}{b} > \frac{c}{d}, \quad \frac{a \cdot d}{b \cdot d} > \frac{c \cdot b}{d \cdot b}, \quad \frac{ab}{bd} > \frac{bc}{bd}, \quad ad > bc. \ \blacksquare$$
3. The following steps can be performed forwards or backwards:
$$\frac{a}{b} < \frac{c}{d}, \quad \frac{a \cdot d}{b \cdot d} < \frac{c \cdot b}{d \cdot b}, \quad \frac{ab}{bd} < \frac{bc}{bd}, \quad ad < bc. \ \blacksquare$$

Fraction Models 1

1. Draw a circle with 2 parts. Shade 1 part one way and 1 part another way. The sum is the number of parts now shaded (2) out of the total 2 parts.
2. Draw a circle with 4 parts. Shade 2 parts one way and 1 part another way. The sum is the number of parts now shaded (3) out of the total 4 parts.
3. Draw a circle with 6 parts. Shade 3 parts one way and 1 part another way. The sum is the number of parts now shaded (4) out of the total 6 parts.
4. Draw a circle with 2 parts. Shade 2 parts one way and then cross out 1 part to denote that part being removed. The difference is the number of shaded parts remaining (1) out of the total 2 parts.

5. Draw a circle with 4 parts. Shade 2 parts one way and then cross out 1 part to denote that part being removed. The difference is the number of shaded parts remaining (1) out of the total 4 parts.
6. Draw a circle with 6 parts. Shade 3 parts one way and then cross out 1 part to denote that part being removed. The difference is the number of shaded parts remaining (2) out of the total 6 parts.

<u>Adding/Subtracting Fractions</u>

1. LCD is 12, so $\frac{1}{4}+\frac{1}{6}=\frac{3}{12}+\frac{2}{12}=\frac{5}{12}$.
2. LCM is 6, so $\frac{1}{3}+\frac{1}{6}=\frac{2}{6}+\frac{1}{6}=\frac{3}{6}=\frac{1}{2}$.
3. LCM is 54, so $\frac{5}{18}+\frac{5}{27}=\frac{15}{54}+\frac{10}{54}=\frac{25}{54}$.
4. Cross multiplication theorem: $\frac{5}{72}+\frac{1}{30}=\frac{5\cdot 30+1\cdot 72}{72\cdot 3}=\frac{222}{216}=\frac{37}{36}$.
5. Cross multiplication theorem: $\frac{78}{104}+\frac{35}{91}=\frac{78\cdot 91+35\cdot 104}{104\cdot 91}=\frac{10738}{9464}=\frac{59}{52}$.
6. Cross multiplication theorem: $\frac{182}{588}+\frac{165}{693}=\frac{182\cdot 693+165\cdot 588}{588\cdot 693}=\frac{223146}{407484}=\frac{23}{42}$.
7. LCD is 12, so $\frac{1}{4}-\frac{1}{6}=\frac{3}{12}-\frac{2}{12}=\frac{1}{12}$.
8. LCM is 6, so $\frac{1}{3}-\frac{1}{6}=\frac{2}{6}-\frac{1}{6}=\frac{1}{6}$.
9. LCM is 54, so $\frac{5}{18}-\frac{5}{27}=\frac{15}{54}-\frac{10}{54}=\frac{5}{54}$.
10. Cross multiplication theorem: $\frac{5}{72}-\frac{1}{30}=\frac{5\cdot 30-1\cdot 72}{72\cdot 30}=\frac{78}{2160}=\frac{13}{360}$.
11. Cross multiplication theorem: $\frac{78}{104}-\frac{35}{91}=\frac{78\cdot 91-35\cdot 104}{104\cdot 91}=\frac{3458}{9464}=\frac{19}{52}$.
12. Cross multiplication theorem: $\frac{182}{588}-\frac{165}{693}=\frac{182\cdot 693-165\cdot 588}{588\cdot 693}=\frac{29106}{407484}=\frac{1}{14}$.

<u>Adding/Subtracting Mixed Numbers</u>

1. $2\frac{1}{2}+1\frac{3}{4}=2\frac{2}{4}+1\frac{3}{4}=3+\frac{5}{4}=3+1\frac{1}{4}=4\frac{1}{4}$.
2. $3\frac{3}{8}+\frac{5}{8}=3+\frac{8}{8}=3+1=4$.
3. $5\frac{7}{10}+2\frac{8}{15}=5\frac{21}{30}+2\frac{16}{30}=7+\frac{37}{30}=7+1\frac{7}{30}=8\frac{7}{30}$.
4. $2\frac{1}{2}-1\frac{3}{4}=2\frac{2}{4}-1\frac{3}{4}=1+\frac{6}{4}-1\frac{3}{4}=\frac{3}{4}$.
5. $3\frac{3}{8}-\frac{5}{8}=2\frac{11}{8}-\frac{5}{8}=2\frac{6}{8}=2\frac{3}{4}$.
6. $5\frac{1}{10}-2\frac{8}{15}=5\frac{3}{30}-2\frac{16}{30}=4\frac{33}{30}-2\frac{16}{30}=2\frac{17}{30}$.

<u>Proofs 2</u>

1. $\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d}{b\cdot d}+\frac{c\cdot b}{d\cdot b}=\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd}$. ■

2. $\frac{a}{b} - \frac{c}{d} = \frac{a \cdot d}{b \cdot d} - \frac{c \cdot b}{d \cdot b} = \frac{ad}{bd} - \frac{bc}{bd} = \frac{ad - bc}{bd}$. ■

Fraction Models 2

1. Draw a rectangle, divide it into 4 vertical bars, then shade 4 of them. Now divide this figure into 2 horizontal bars and shade 1 of them in a different manner than the first. This gives 3 squares shaded both ways, which is your numerator, and there are 8 squares total, which is your denominator.
2. Draw a rectangle, divide it into 5 vertical bars, then shade 2 of them. Now divide this figure into 3 horizontal bars and shade 2 of them in a different manner than the first. This gives 4 squares shaded both ways, which is your numerator, and there are 15 squares total, which is your denominator.
3. Draw a rectangle, divide it into 6 vertical bars, then shade 5 of them. Now divide this figure into 5 horizontal bars and shade 4 of them in a different manner than the first. This gives 20 squares shaded both ways, which is your numerator, and there are 30 squares total, which is your denominator.

Multiplying Fractions

1. $\frac{3}{2} \times \frac{2}{15} = \frac{3}{1} \times \frac{1}{15} = \frac{1}{1} \times \frac{1}{5} = \frac{1}{5}$.
2. $\frac{3}{14} \times \frac{70}{20} = \frac{3}{14} \times \frac{7}{2} = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$.
3. $\frac{21}{28} \times \frac{40}{66} = \frac{7}{28} \times \frac{40}{22} = \frac{1}{4} \times \frac{20}{11} = \frac{1}{1} \times \frac{5}{11} = \frac{5}{11}$.

Multiplying Mixed Numbers

1. FOIL: $1\frac{1}{2} \times 2\frac{1}{2} = \left(1 + \frac{1}{2}\right)\left(2 + \frac{1}{2}\right) = 2 + \frac{1}{2} + \frac{2}{2} + \frac{1}{4} = 3 + \frac{2}{4} + \frac{1}{4} = 3\frac{3}{4}$.
2. Improper Fraction: $2\frac{2}{3} \times 1\frac{1}{3} = \frac{8}{3} \times \frac{4}{3} = \frac{32}{9} = 3\frac{5}{9}$.
3. FOIL: $6\frac{1}{4} \times 3\frac{3}{5} = \left(6 + \frac{1}{4}\right)\left(3 + \frac{3}{5}\right) = 18 + \frac{18}{5} + \frac{3}{4} + \frac{3}{20} = 18 + \frac{72}{20} + \frac{15}{20} + \frac{3}{20} = 18 + \frac{90}{20} = 18 + 4\frac{1}{2} = 22\frac{1}{2}$.

Dividing Fractions

1. Let $n = \frac{3}{8} \div \frac{8}{3}$. Then by definition, $\frac{8}{3}n = \frac{3}{8}$. Multiply both sides by $\frac{3}{8}$ (the multiplicative inverse of $\frac{8}{3}$ to get $n = \frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$.
2. $\frac{\frac{5}{12}}{\frac{3}{4}} \times \frac{\frac{4}{3}}{\frac{4}{3}} = \frac{\frac{5}{12} \times \frac{4}{3}}{1} = \frac{5}{12} \times \frac{4}{3} = \frac{5}{3} \times \frac{1}{3} = \frac{5}{9}$.
3. $\frac{30}{35} \div \frac{99}{28} = \left(\frac{30}{35} \div \frac{99}{28}\right) \times \left(\frac{99}{28} \times \frac{28}{99}\right) = \frac{30}{35} \div \frac{99}{28} \times \frac{99}{28} \times \frac{28}{99} = \frac{30}{35} \times \frac{28}{99} = \frac{30}{5} \times \frac{4}{99} = \frac{6}{1} \times \frac{4}{99} = \frac{2}{1} \times \frac{4}{33} = \frac{8}{33}$.

Properties of Multiplication

1. Closure holds since $a^2b^2 = (ab)^2$ and all the perfect squares appear. Commutative and Associative hold for all multiplication of real numbers. Identity holds since 1 is in the set. Inverse does not hold since no reciprocals are present.
2. Closure does not hold because any number times its reciprocal is 1, and 1 is not in the set. Commutative and Associative hold for all multiplication of real numbers. Identity does not hold since 1 is not in the set. Inverse holds since all reciprocals are present.

3. This set is the set of all positive fractions. Closure holds since you are simply multiplying numerators and denominators from the closed set of natural numbers. Commutative and Associative hold for all multiplication of real numbers. Identity holds since 1 is in the set $\left(\frac{1}{1}\right)$. Inverse holds since all reciprocals are present (switch the roles of a and b).

Decimal Properties

1. (a) $12.035 = 12 + \frac{0}{10} + \frac{3}{100} + \frac{5}{1000}$ and $12.035 = 12 + \frac{35}{1000}$. (b) "Twelve and thirty-five thousandths". (c) 2 in units, 5 in thousandths.
2. (a) $0.1586 = 0 + \frac{1}{10} + \frac{5}{100} + \frac{8}{1000} + \frac{6}{10000}$ and $0.1586 = 0 + \frac{1586}{10000}$. (b) "Zero and one thousand five hundred eighty-six ten-thousandths". (c) 1 in tenths, 6 in ten-thousandths.
3. (a) $920.74 = 920 + \frac{7}{10} + \frac{4}{100}$ and $920.74 = 920 + \frac{74}{100}$. (b) "Nine hundred twenty and seventy-four hundredths". (c) 9 in hundreds, 0 in thousandths (need to add a zero to see this).
4. Example: $3.900 = 3 + \frac{9}{10} + \frac{0}{100} + \frac{0}{1000} = 3 + \frac{9}{10} = 3.9$.

Decimals to Fractions

1. $0.625 = \frac{625}{1000} = \frac{25}{40}$.
2. $0.5625 = \frac{5625}{10000} = \frac{9}{16}$.
3. $0.0175 = \frac{175}{10000} = \frac{7}{400}$.

Fractions to Decimals

1. $\frac{6}{25}$ is simplified and $25 = 5^2$, so it terminates. We want to get 10^2 in the denominator since 5^2 is already there, so we multiply by 2^2: $\frac{6 \cdot 2^2}{5^2 \cdot 2^2} = \frac{24}{10^2} = 0.24$.
2. $\frac{2}{9}$ is simplified and $9 = 3^2$, so it repeats. By long division, this is $0.\overline{2}$.
3. $\frac{13}{40}$ is simplified and $40 = 2^3 \cdot 5$, so it terminates. We want to get 10^3 in the denominator since 2^3 is already there, so since we already have 5, we multiply by 5^2: $\frac{13 \cdot 5^2}{2^3 \cdot 5 \cdot 5^2} = \frac{325}{10^3} = 0.325$.
4. $\frac{57}{75} = \frac{19}{25}$ and $25 = 5^2$, so it terminates. We want to get 10^2 in the denominator since 5^2 is already there, so we multiply by 2^2: $\frac{19 \cdot 2^2}{5^2 \cdot 2^2} = \frac{76}{10^2} = 0.76$.
5. $\frac{21}{99} = \frac{7}{33}$ and $33 = 3 \cdot 11$, so it repeats. By long division, this is $0.\overline{21}$.
6. $\frac{101}{144}$ is simplified and $144 = 2^4 \cdot 3^2$, so it repeats. By long division, this is $0.7013\overline{8}$.

Operations on Decimals

1. $1.237 + 3.249 = 4.486$.
2. $0.007 + 15.1937 = 15.2007$.

3. $5.214 - 0.763 = 4.451$.
4. $3.34 - 1.3491 = 1.9909$.
5. $3.22 \times 5.7 = \frac{322}{10^2} \times \frac{57}{10} = \frac{18354}{10^3} = 18.354$.
6. $40.001 \times 8.91 = 356.40891$.
7. $122.5 \div 16 = 7.65625 \approx 7.66$.
8. $659 \div 5.7 = \frac{659}{5.7} \times \frac{10}{10} = \frac{6590}{57} = 6590 \div 57 \approx 115.61$.
9. $0.001 \div 8.91 = 0.1 \div 891 = 1.12 \times 10^{-4}$ (rounding here and using scientific notation)
10. Look at the method used to solve problem 5. When we multiplied our denominators, we effectively added the number of decimal places present in each number and used that for the number of decimal places in our answer.
11. When we add a ".0", we are effectively converting our remaining units into tenths are partitioning those instead of the units. When we add additional zeros, we are converting further as needed into smaller units.
12. Look at the method used to solve problem 8. When we multiplied the dividend and divisor by 10, we effectively multiplied and divided by 10, which did not change the problem.

Rounding Decimals

1. Round up to 1.9.
2. Round up to 0.83.
3. Round up (with some regrouping) to 12.800.

Comparing Decimals

1. All digits are the same until $2 > 0$ in the thousandths place. Thus, $1.142 > 1.14$.
2. $1.\overline{59} = 1.595959...$ and $1.\overline{590} = 1.590590....$ All digits are the same until $5 > 0$ in the thousandths place. Thus, $1.\overline{59} > 1.\overline{590}$.
3. $0.\overline{63} = 0.636363...$ and $0.\overline{637} = 0.637637....$ All digits are the same until $6 < 7$ in the thousandths place. Thus, $0.\overline{63} < 0.\overline{637}$.